Myopia in dynamic spatial games

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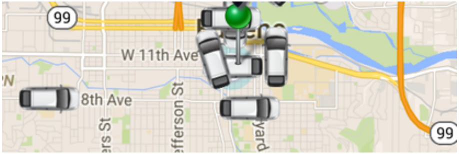


The other Uber drivers have boxed me in... I won't get a rider unless I move or wait for them to get pings first.

Summary

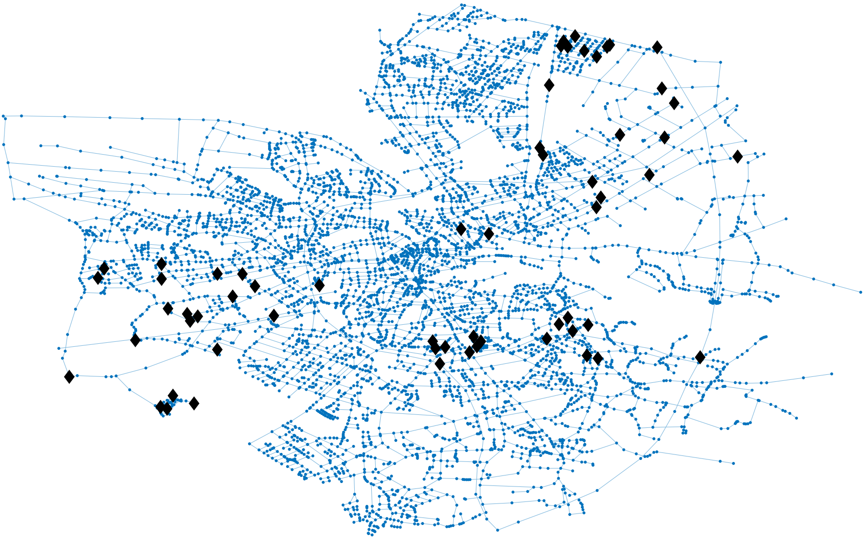
In the paper, we . . .

1. develop a dynamic model of spatial competition and design an experiment to analyze behavior in the model.
2. present the prevalence of myopic best responding in experiment results.
3. develop agent-based models to test the efficiency consequences of myopically best responding drivers.

Model: Environment and agents

- The transportation network G is given by the pair $G = (\mathcal{N}, A)$.
  - $\mathcal{N}$ is the set of intersections in the transportation network with N elements.
  - $A \in [0, \infty]^{N \times N}$ is a weighted adjacency matrix. If there is a road between nodes m and n, $a_{mn} \in \mathbb{R}_{\geq 0}$ denotes the distance or weight. Otherwise, $a_{mn} = \infty$.
- Node n's set of neighbors $\mathcal{B}_n$ consists of the nodes that n shares an edge with: $\mathcal{B}_n = \{m \in \mathcal{N} : a_{nm} < \infty\}$.
- Passengers are distributed across the locations in G according to the function $\theta : \mathcal{N} \to \mathbb{R}_+$.
- $\mathcal{I}$ is the set of Lyft drivers in this game ($|\mathcal{I}| = I$).
- A *spatial allocation* s_t is an I-tuple that records the current positions of the Lyft drivers on the locations of G in period t:

$$(s_{t,1}, s_{t,2}, \ldots, s_{t,I}) \in \prod_{i \in \mathcal{I}} \mathcal{N} = \mathcal{S}.$$



Spatial allocation of 60 drivers on transportation network of Oldenburg, Germany. Black diamonds are Lyft drivers.

Model: Agent incentives

- The multiplicity function records the cardinality of the set of Lyft drivers who are equidistant from a particular location,

$$\psi(n, s_t) = |\arg\min_{i \in \mathcal{I}} \{d(n, s_{t,i})\}|, \tag{1}$$

  where $d(\cdot)$ finds shortest distance.

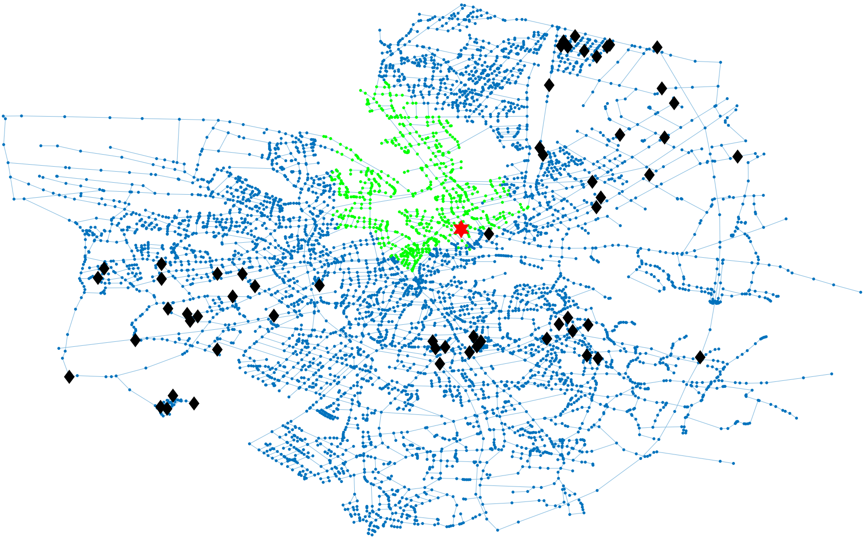
- In each period t, we use a Voronoi diagram $\mathrm{Vor}(s_t)$ to calculate each driver's market share, which is related to her Voronoi cell $V_i(s_t)$,

$$V_i(s_t) = \{n \in \mathcal{N} \mid d(n, s_{t,i}) \leq d(n, s_{t,j}), \forall j \neq i\} \tag{2}$$

- At period t in the game, the market share of driver i is:

$$\pi_i(s_t) = \sum_{n \in V_i(s_t)} \frac{\theta(n)}{\psi(n, s_t)}. \tag{3}$$

- Agents moves sequentially, each maximizing $\sum_t \pi_i(s_t)$.



Green nodes belong to Voronoi Region of Red star Lyft driver. Black diamonds are other Lyft drivers.

Model: Efficiency

- Calculate the average distance between a passenger and her nearest Lyft driver $\bar{d}(s_t)$ of the spatial allocation s_t in period t as:

$$\bar{d}(s_t) = \frac{1}{\sum_{n \in \mathcal{N}} \theta_n} \left(\sum_{n \in \mathcal{N}} \theta_n \min_{i \in \mathcal{I}} \{ d(n, s_{t,i}) \} \right). \tag{4}$$

- Define an optimal spatial allocation s^* as one which minimizes (4):

$$s^* \in \arg\min_{s_t \in \mathcal{S}} \bar{d}(s_t). \tag{5}$$

- Define the spatial inefficiency of a spatial allocation s_t as

$$\xi(s_t) = \frac{\bar{d}(s_t) - \bar{d}(s^*)}{\bar{d}(s^*)}. \tag{6}$$

Approximately optimal allocation of 60 Lyft drivers in Oldenburg.

Experiment design

5 players play a game on a 21x21 grid. Instructions video

- Grid surrounded by static computer players.

One-by-one, players have opportunity to move one square.

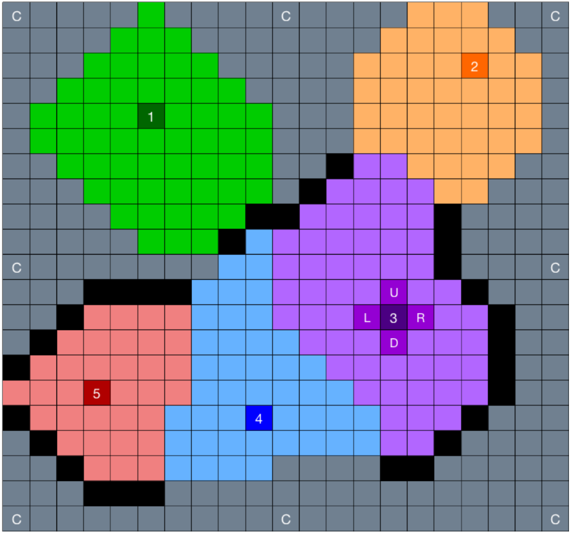
- Experiment session runs for 90 minutes.
- Payments proportional to average area size over the experiment.
    - A player's area is the number of squares closest to her (by ℓ_1 norm).

Experiment software:

- Main console (projected) shows the current state of the game.
- Calculator software (on each PC) facilitates reasoning.
    - Average: 275 calculations. Max: 786.

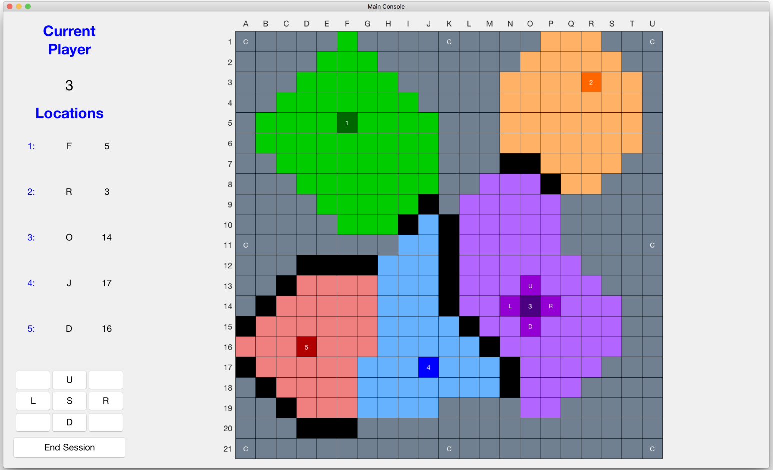
Experiment run in June, 2017 at UW-Madison BRITE Lab.

Design: Game board initialization



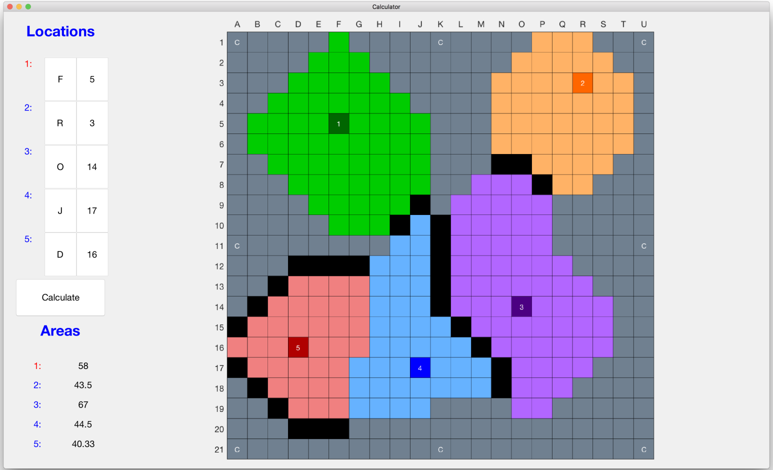
Initial positions of five players and their areas. Example session video

Design: Main console software (projected)



Includes turn indicator, current locations, and movement choices.

Design: Calculator software (on each player PC)



Input coordinates of players and see areas and resulting grid.

Results

In **18 sessions**, we observed **2178 moves** and **24765 calculations**.

Moves: Move rankings non-MBR moves SHO moves

Over 60% of moves maximized current flow playoff (MBR).

- Of the 863 non-MBR moves, 556 plausibly attributable to error.
- 307 moves suggestive of higher-order reasoning (SHO).

Calculations: Player positioning

In 82% of calculations, opponents were positioned in their locations.

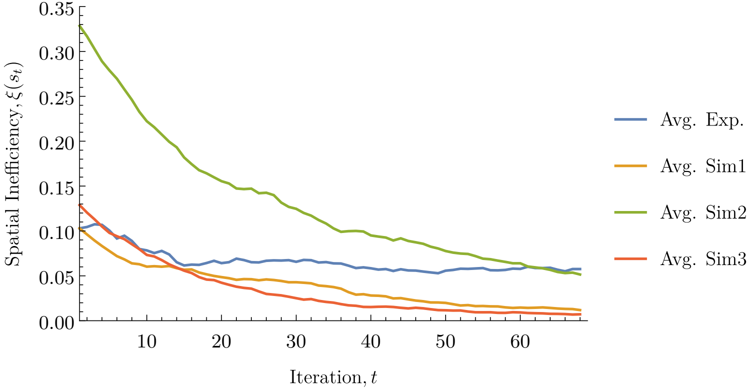
In 92%, the calculating player was within one square of her location.

Payoffs: Regression results

SHO moves: no effect on payment. # MBR moves: positive effect.

- Learning may push players towards MBR.

Application: The experiment environment

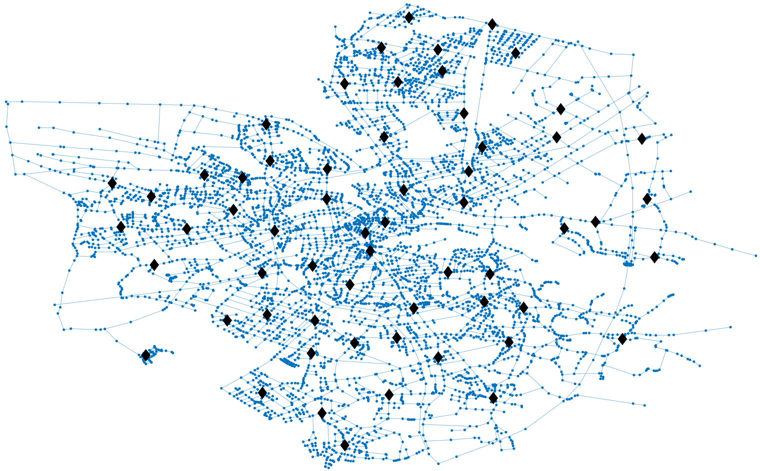


Spatial inefficiency in experiment and simulations

Experimental behavior improved efficiency, but not as much as in simulations with pure MBR players.

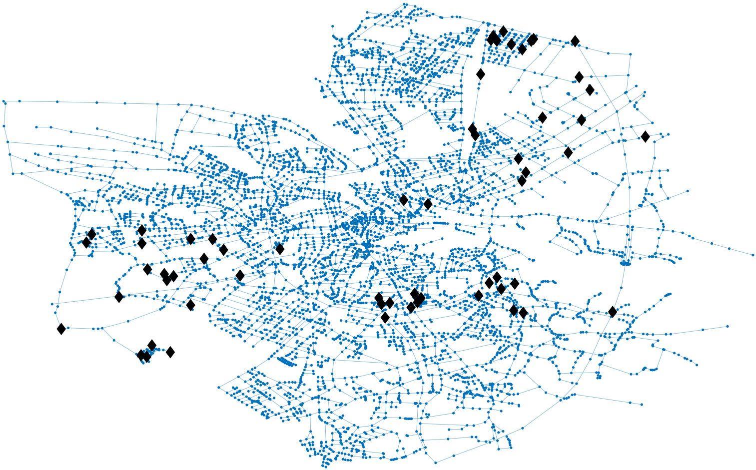
Application: Oldenburg

We also ran MBR simulations with 60 drivers on the actual transportation network of Oldenburg, Germany (6,105 nodes and 7,029 edges):



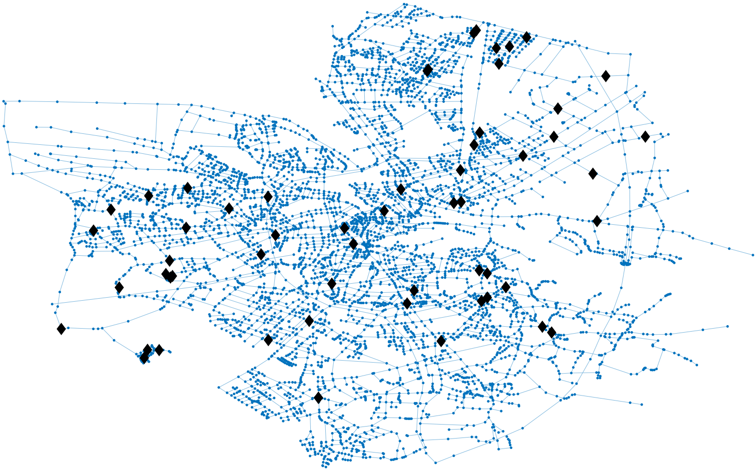
Transportation network in Oldenberg with approx. optimal allocation

Application: Oldenburg (Initial)



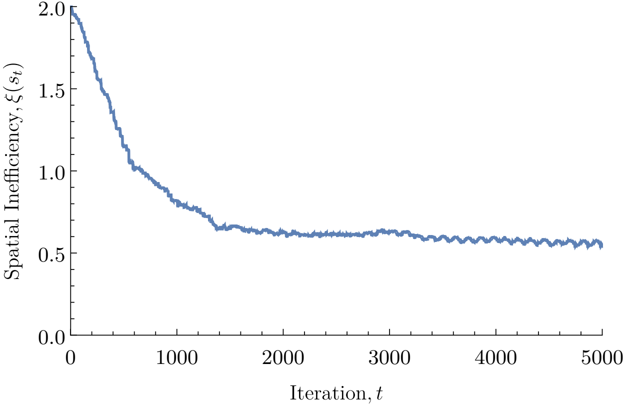
Random initial allocation of 60 drivers in Oldenburg; $\xi(s_1) = 2.02$

Application: Oldenburg (Final)



Final allocation of 60 drivers in Oldenburg; $\xi(s_{5000}) = 0.55$

Application: Oldenburg (MBR and Efficiency)



Spatial inefficiency along dynamic path of simulation with $T = 5000$

MBR behavior improved efficiency, but not approaching the optimal allocation. Can do better by fixing peripheral players.

Conclusion

We have observed a prevalence of myopic best responding in an experiment designed to test behavior in a dynamic spatial game.

We have argued that this behavioral insight facilitates predictive agent-based models.

Our agent-based models suggest that MBR behavior has generally positive consequences for efficiency.

- Uber and Lyft may want to show drivers their competitors' locations to facilitate myopic best responding amongst its their idle drivers.

Potential future work:

- Is MBR a valid assumption in higher-stakes games with non-reversible choices, such as firm location decisions?
- How do the efficiency consequences of the MBR dynamic depend on the structure of the network?

Discussion: Challenges of static analyses

Static games of spatial competition

- Each firm simultaneously chooses its location to maximize profits. Nash equilibrium where no firm has profitable deviation.

Key challenges to predictions from static spatial games:

1. Equilibrium multiplicity
2. Equilibrium non-existence
3. Intractability

May also have limited predictiveness in dynamic settings.

Discussion: Agent-based models

The underlying complexity of spatial competition that impedes static equilibrium analyses may also result in players behaving predictably.

If behavior is predictable, we can yield predictions through agent-based models (ABM). An ABM is a computational model for simulating the interactions of autonomous agents to assess their effects on the system.

Given the prevalence of myopic best responding that we observe experimentally, we think it is reasonable to pursue predictions in spatial environments through ABM with MBR agents.

- Next, we consider applications of ABM in our experiment environment and on a realistic transportation network.

A fixed point under the MBR dynamic is also a Nash equilibrium in the analogous static game, so our ABM may also be used to find these.

Distribution of move MBR rankings

	All moves		After 10 mins	
MBR ranking	Freq.	%	Freq.	%
1	1315	60	1103	63
2	439	20	339	20
3	229	11	174	10
4	126	6	74	4
5	69	3	50	3

Higher MBR-ranked moves are selected more often.

Partitioning the 863 non-MBR moves

	MBR not calculated 535	**MBR calculated** 328
Choice not calculated	325	21
No calculations	(205)	-
Some calculations[1]	(120)	-
Choice calculated	210	307
Chose HSCM	(126)	-
Did not choose HSCM	(84)	-

All but 307 non-ZCV1 moves are attributable to error or apathy.

Number of SHO moves by player

# SHO Moves	# Players	% Players	% Total SHO
0	19	21	0
1	15	17	5
2	11	12	7
3	9	10	9
4	11	12	14
5	6	7	10
6	4	4	8
7	4	4	9
8	3	3	8
9	2	2	6
≥ 10	6	7	24

About 20% of players accounted for 50% of SHO moves.

Score differences for non-MBR moves

	SHO %	**non-SHO %**
Score Diff > 1	22	49
Score Diff ≤ 1	78	51
Score Diff $< .5$	33	14
Score Diff $< .25$	17	6

SHO moves tended to have low score differences—players appear to go on intuition or higher-order reasoning when scores or close.

Or, SHO moves might result from players failing to notice small differences in MBR scores. . .

Player positioning in calculations

	Freq.	**%**
Total	24765	-
Opponents in place	20341	82
Move option	18869	76

Positioning of opponents in calculations

Distance	**Freq.**	%	**Of which opponents in place**
0	12046	49	9546
1	10733	43	9323
2	1205	5	957
3	360	2	253
≥ 4	421	2	262

Distribution of player distances from their iteration position in calculations

Regression results

Model:	(1)	(2)	(3)	(4)	(5)
# Turns	-0.171	0.0218	-0.0288	-0.179	-0.00974
	(0.104)	(0.0871)	(0.0851)	(0.103)	(0.0852)
# MBR	0.441**			0.391*	
	(0.158)			(0.158)	
# SHO		-0.221			-0.375
		(0.234)			(0.234)
# Calcs.			0.0137*	0.0112	0.0162*
			(0.00605)	(0.00596)	(0.00619)
N	90	90	90	90	90
adj. R^2	0.062	-0.013	0.034	0.088	0.051

Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$

OLS: Differences (in thousandths) from mean score (by player number)

#MBR moves has a positive effect on score. #SHO is insignificant.